Reduction of optical forces exerted on nanoparticles covered by scattering cancellation based plasmonic cloaks

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Scattering cancellation approach has been recently proposed as a promising route to design invisibility cloaks. However, reduced observability of an object is just one of the potential applications of this technique. In this paper, we investigate the possibility to reduce optical forces exerted on a given nanoparticle by covering it with a properly designed plasmonic cloak. We show, in fact, that conditions similar to those used to make spherical and cylindrical nanoparticles invisible to the electromagnetic field by using the scattering cancellation approach, may be straightforwardly applied also to minimize both gradient and scattering optical forces exerted by the illuminating radiation on the same covered nanoparticles. These results are then validated through full-wave simulations, properly considering both dispersion and losses of the plasmonic materials used to design the cloaks. We also extend our speculations to the case of optical torques exerted on spheroidal and cylindrical Rayleigh particles, deriving the conditions to obtain stable equilibrium positions. This investigation leads to the anomalous result that the usual unstable equilibrium positions of uncovered particles may result stable ones when properly designing the particle cover. Finally, in order to apply the proposed theoretical speculations to more complex cases, we derive the conditions for minimizing optical forces exerted on a cloaked Rayleigh particle placed above a dielectric half space. These results may find interesting applications in different fields of nanotechnology.

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I. INTRODUCTION

Scattering cancellation technique has been recently proposed as one of the possible routes to synthesize invisibility cloaking devices.[1–11] Following this approach, the scattered field from a given object may be drastically reduced by employing a properly designed conformal plasmonic cover, exhibiting an effective relative electric permittivity smaller than unity.[1] Such an engineered, in fact, produces a scattered field which compensates the one of the bare object, thus making the scatterer invisible. Even if initially formulated only for canonical geometries, scattering cancellation approach has been recently extended also to arbitrarily shaped scatterers, showing how a properly designed plasmonic shell may be successfully used to cloak objects characterized by a strongly anisotropic electromagnetic response.[5] What is more, the inherently nonresonant nature of this cloaking technique makes it rather robust to slight variations in geometrical/electrical parameters of the initial design, ensuring also acceptable performances in terms of operating bandwidth.[6] Such promising properties, encouraged some groups to work also on the synthesis of actual cloaks, working both at microwave and optical frequencies.[7–11]

So far, most of the research efforts concerning scattering cancellation have been focused in making a cloaked object nearly transparent to the illuminating electromagnetic radiation. As clearly demonstrated in Refs. 1–3, this phenomenon is inherently related to the dispersive nature of the plasmonic material used for the cloak, which, in the long-wavelength limit, is characterized by a locally negative polarizability compensating the electric dipole scattering contribution due to the object.

In the Rayleigh approximation, however, if the object dimensions approach the nanoscale, optical forces come into play, and their contribution cannot be neglected any more. As early speculated in Ref. 12, metamaterials and plasmonic media may allow to control such forces through their inherent dispersive behavior. This implies, for instance, that it is possible to manipulate light forces simply by changing the material characteristics or the operating frequency. Typically, light forces on electrically small particles are explained in terms of gradient and scattering forces.[13] The former are directly related to the interaction between the external field and the induced dipole, which is drawn by the field-intensity gradients. The latter are related to the momentum transfer between the scattered field and the illuminated particle, being proportional to the Poynting vector.[13,14] Since both these forces are inherently associated to the scattering properties of the object, it is interesting to study their behavior when a cloak placed around a nanoparticle causes the reduction, or even the suppression, of the total scattered field.

The aim of this paper is to combine optical force calculation to the electromagnetic cloaking formulation, in order to obtain further physical insights and explore new possibilities for the manipulation of nanostructures with promising results for several applications in current nanotechnology. The structure of the paper is as follows. In Sec. II, starting from the relation between optical forces exerted on a covered nanoparticle and its polarizability, we derive the conditions under which it is possible to minimize the forces acting on spherical and cylindrical Rayleigh particles, illuminated by a polarized light. In Sec. III, we extend the formulation reported in Sec. II to the case of optical torques exerted on elongated particles, deriving conditions for stable and unstable equilibrium positions. Finally, in Sec. IV, in order to apply the proposed idea to a more complex configuration, we derive the conditions for minimizing optical forces exerted on a cloaked Rayleigh particle placed above a dielectric half space.

II. OPTICAL FORCES ON RAYLEIGH PARTICLES

Optical forces acting on a nanoparticle can be derived from the conservation of the linear momentum introducing Maxwell's stress tensor.[13] Without loss of generality, we may consider an object made of an isotropic homogeneous dielectric, surrounded by vacuum and illuminated by a time harmonic electromagnetic field (excitation of the kind $e^{j\omega t}$ is assumed throughout the paper). Neglecting the particle motion and assuming mass density and optical properties of the object to be constant with respect to the exerted pressure, the time average force $\langle \mathbf{F} \rangle$ exerted on the particle is given by[13,15]

$$\langle \mathbf{F} \rangle = \frac{1}{4}\mathfrak{Re}[\alpha_e] \nabla |\mathbf{E}_0|^2 + \frac{\sigma_t}{2c}\mathfrak{Re}[\mathbf{E}_0 \times \mathbf{H}_0^*] + \frac{\sigma_t}{2k_0}\mathfrak{Im}[\varepsilon_0(\mathbf{E}_0^* \cdot \nabla)\mathbf{E}_0], \quad (1)$$

where $\mathfrak{Re}[\circ]$ and $\mathfrak{Im}[\circ]$ are real and imaginary parts, respectively, $[\circ]^*$ complex conjugation, $k_0 = \omega/c$ the free-space wave number, σ_t the total scattering cross-section (SCS), and α_e the electric polarizability of the object. In the Rayleigh approximation, any given object with anisotropic scattering properties may be described by its polarizability tensor $\underline{\boldsymbol{\alpha}}_e$.[16,17] For highly symmetrical small scatterers, the polarizability tensor may be assumed as a scalar quantity α_e while, for more complex geometries, a similar approach may be still used, assuming the polarizability tensor to be uniaxial in a convenient coordinate system oriented along the three main axes of the scatterer.[18] By means of the optical theorem[19] we can straightforwardly express σ_t in terms of the electric polarizability of the object as

$$\sigma_t = \frac{k_0}{\varepsilon_0}\mathfrak{Im}[\alpha_e]. \quad (2)$$

Therefore, in Eq. (1) we distinguish three terms, related to the acting forces on the particle: the first one is the *gradient force*, the second one is the scattering force or *radiation pressure*, and the third one, which is generally neglected in the case of electrically small objects,[20] has been recently associated to the time-averaged spin density of a transverse electromagnetic field.[15]

According to Eq. (1), by nullifying the complex electric polarizability α_e (both real and imaginary parts), we are able to contemporary minimize all the force contributions acting on Rayleigh scatterers. In the following, we show how it is possible to achieve such condition by using scattering cancellation approach, in the general case of lossy materials described by proper dispersion models.

A. Spherical nanoparticles

Let us consider, as a first example, the case of a *spherical particle* of radius a and permittivity function $\varepsilon(\omega) = \varepsilon_0\varepsilon_r(\omega)$. Due to the symmetry of the problem, the electric polarizability does not depend on the polarization of the electric field. In the limit $k_0 a \ll 1$, the electric polarizability $\alpha_e^s(\omega)$ is given by[15,21]

$$\alpha_e^s(\omega) = \frac{\alpha_e^0(\omega)}{1 - j\dfrac{k_0^3\alpha_e^0(\omega)}{6\pi\varepsilon_0}}, \quad \text{with} \quad \alpha_e^0(\omega) = 3\varepsilon_0 V\frac{\varepsilon_r(\omega) - 1}{\varepsilon_r(\omega) + 2}, \quad (3)$$

where the radiation-reaction term (due to the interaction of the dipole with its own radiated field) has been introduced, in order to satisfy the optical theorem.[13,15] It is worth noticing that optical force formulation, derived through Maxwell's stress tensor, is fully consistent with the approach based on the Lorentz force calculation.[22]

From Eq. (3), it may be easily derived that an electrically small sphere, made of a regular dielectric material with $\varepsilon_r > 1$, will always experience an acting optical force, according to Eq. (1). Let us assume, now, that the scatterer is made of a material whose relative permittivity $\varepsilon_r(\omega)$ follows Drude dispersion model as

$$\varepsilon_r(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega - j\nu_c)}, \quad (4)$$

being ω_p the plasma frequency, ε_∞ the upper frequency permittivity limit, and ν_c the damping factor. As it results from the expression of α_e^s, the complex polarizability exhibits a resonant behavior at $\omega = \omega_r$, where the plasmonic resonance occurs for $\varepsilon_r(\omega_r) = -2$, thus implying that $\mathfrak{Re}[\alpha_e^s(\omega_r)] = 0$ while $\mathfrak{Im}[\alpha_e^s(\omega_r)]$ reaches its maximum. As it is well know, in this case, the radiation pressure term is enhanced by the plasmonic nature of the material at the resonance frequency. On the other hand, being the real part of α_e^s almost zero at $\varepsilon_r(\omega_r) = -2$, the gradient force term is significantly reduced, even if $\nabla|\mathbf{E}_0|^2 \neq 0$, as it may be generally assumed. Only under plane-wave illumination, in fact, the first and the last terms of Eq. (1) are identically zero,[15] being the radiation pressure, which is proportional to the Poynting vector, the only force contribution acting on the scatterer.

Now, if we cover the scatterer with a plasmonic shell of radius b and relative permittivity $\varepsilon_c(\omega)$, the overall polarizability α_e^{cs} of the rigid covered particle, may be consistently expressed as[23,24]

$$\alpha_e^{cs}(\omega) = \frac{\alpha_e^0(\omega)}{1 - j\dfrac{k_0^3\alpha_e^0(\omega)}{6\pi\varepsilon_0}},$$

$$\alpha_e^0(\omega) = 4\pi b^3\varepsilon_0 \times \frac{b^3(\varepsilon_c - 1)(2\varepsilon_c + \varepsilon_r) - a^3(2\varepsilon_c + 1)(\varepsilon_c - \varepsilon_r)}{b^3(\varepsilon_c + 2)(2\varepsilon_c + \varepsilon_r) - 2a^3(\varepsilon_c - 1)(\varepsilon_c - \varepsilon_r)}, \quad (5)$$

where the frequency dependence of the permittivities has been omitted for sake of brevity. In the limit case $\varepsilon_c \to 1$, we obtain the polarizability of the bare sphere $\alpha_e^s(\omega)$, otherwise, according to theory developed in Ref. 1, for any given relative permittivity ε_r of the inner particle, it is possible to determine a certain range of permittivity values for the cover material, which allow nullifying contemporarily both real and the imaginary parts of the complex polarizability $\alpha_e^{cs}(\omega)$.

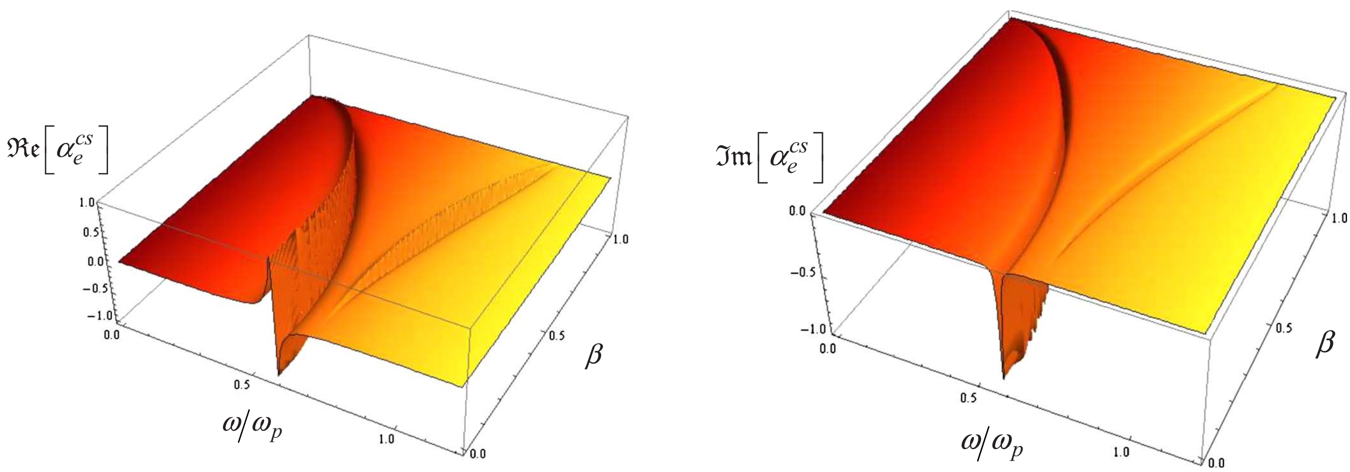

FIG. 1. (Color online) Complex polarizability of a dielectric electrically small spherical particle covered with a plasmonic coating whose permittivity follows Drude dispersion. Real and imaginary parts of the polarizability are normalized to their relative maximum values.

In order to clarify these aspects, let us consider an example. The inner core of a coated spherical particle is made of a regular dielectric with $\varepsilon_r=2$ while the external shell is made of a Drude-type dispersive and lossy material. According to Eq. (5), for a certain value of the permittivity $\varepsilon_c(\omega_0)$ at the design frequency ω_0, we may find an optimal ratio between the radii $\beta=a/b$, assuring that $\alpha_e^{cs}(\omega_0)\approx 0$, thus implying that $\Re\mathfrak{e}[\alpha_e^{s}(\omega_0)]=0$, while keeping the imaginary part extremely low. In Fig. 1, we show the complex polarizability as a function of the normalized frequency ω/ω_p and β with $\nu_c=10^{-2}\omega_p$.

From these plots, we note that there is a frequency region in the dielectric plane described by $\varepsilon_c(\omega)$ and ε_r, in which the condition $\alpha_e^{cs}(\omega_0)\approx 0$ is satisfied (in Fig. 2 we show one of the zero occurrences for $\beta\approx 0.67$).

Polarizability cancellation effect is certainly affected when increasing losses, that is when varying the collision frequency ν_c in the model. However, it is worth noticing that, being the scattering cancellation approach based on a nonresonant phenomenon, we can reasonably assume that losses are generally sufficiently small in the frequency range of interest. Nevertheless, even considering moderately higher losses, it is still possible to obtain the desired behavior, as shown in Fig. 3, where a dependence $\nu_c=10^{-\sigma}\omega_p$ for the collision frequency is assumed.

B. Cylindrical nanoparticles

The proposed formulation is certainly consistent also when describing two-dimensional configurations, that is when considering, for instance, nanorods and wires with electrically small cross-sections. In this case, it is possible to evaluate the acting forces simply by introducing the proper polarizability,[13] according to the previous analysis. Let us consider, then, the case of a circular cylinder of radius a, infinitely extended along its symmetry axis (Fig. 4). Referring to the main polarizations of the impinging field, we can define two different polarizabilities $\alpha_e^{\parallel}(\omega)$ and $\alpha_e^{\perp}(\omega)$, depending on whether the illuminating electric field is parallel or perpendicular to the cylinder axis $(\hat{z})$, respectively.

For an electrically small cylinder with permittivity $\varepsilon(\omega)=\varepsilon_0\,\varepsilon_r(\omega)$, polarizability $\alpha_e^{\parallel}(\omega)$ may be expressed through a series expansion of the zeroth order Mie scattering coefficient as[25]

$$\alpha_e^{\parallel}(\omega)=\frac{\alpha_e^{p}(\omega)}{1-j\dfrac{k_0^2}{4\varepsilon_0}\alpha_e^{p}(\omega)},\quad \alpha_e^{p}(\omega)=\pi a^2\varepsilon_0[\varepsilon_r(\omega)-1] \tag{6}$$

while for the orthogonal polarization $\alpha_e^{\perp}(\omega)$ can be similarly written as[26]

$$\alpha_e^{\perp}(\omega)=\frac{\alpha_e^{o}(\omega)}{1-j\dfrac{k_0^2}{4\varepsilon_0}\alpha_e^{o}(\omega)},\quad \alpha_e^{o}(\omega)=\pi a^2\varepsilon_0\frac{\varepsilon_r(\omega)-1}{\varepsilon_r(\omega)+1}. \tag{7}$$

Looking at expressions (6) and (7), it follows that also in this case, for any $\varepsilon_r\geq 1$, the object experiences an acting force in

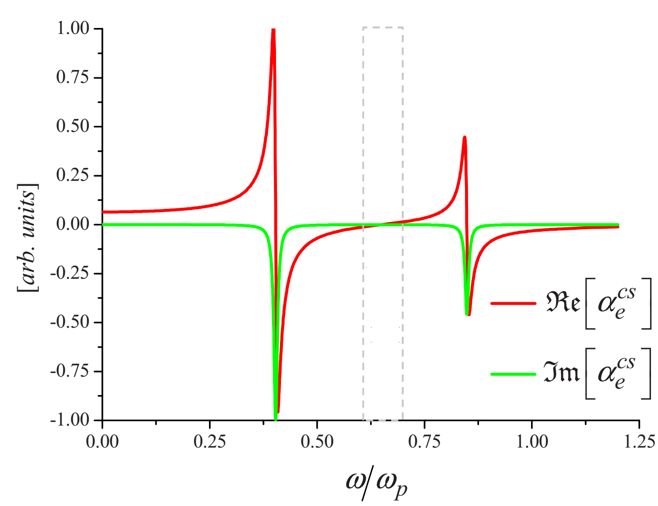

FIG. 2. (Color online) Complex polarizability (normalized to its maximum value) for a dielectric electrically small spherical particle covered with a plasmonic coating, whose permittivity follows Drude dispersion for $\beta\approx 0.67$. The shadowed area indicates the frequency region in which the complex polarizability is almost zero.

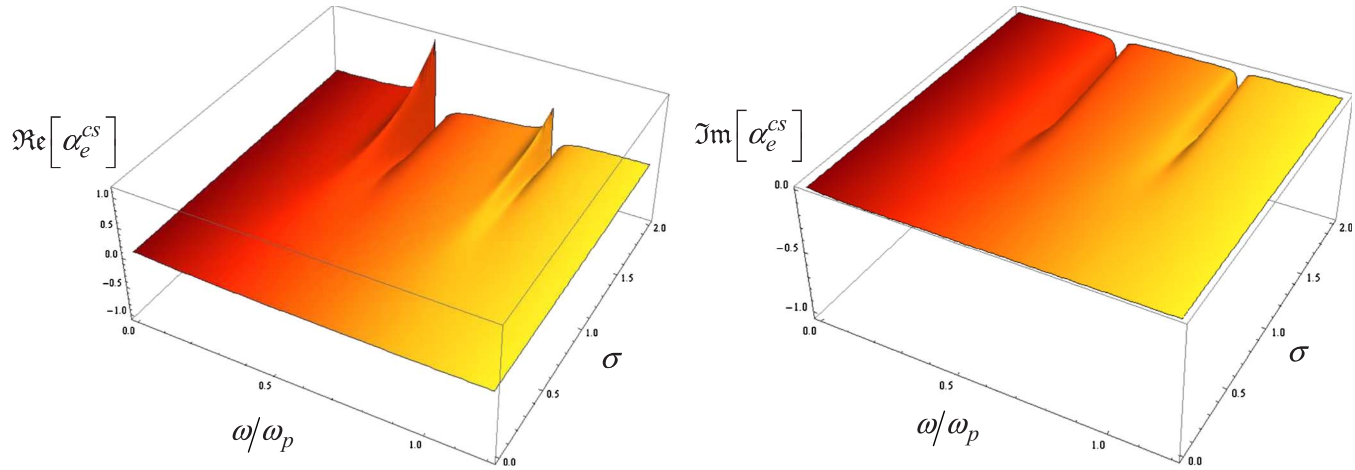

FIG. 3. (Color online) Normalized complex polarizability for a dielectric electrically small spherical particle covered with a plasmonic coating, whose permittivity follows Drude dispersion, for $\beta\approx 0.67$ and assuming losses to vary according to $\nu_c = 10^{-\sigma}\omega_p$.

both the polarizations. If the cylinder is made of a plasmonic material, $\alpha_e^{\perp}$ shows a resonant behavior similar to the one of a small sphere, this time with the material resonance occurring at $\varepsilon_r(\omega_r) = -1$. It is worth noticing that, even if the cylinder is assumed to be infinitely long, the approach still applies for finite-length wires, provided that the effective length L of the object is comparable to the operating wavelength.[27]

Again, when coating the cylinder with an external shell of radius b, as reported in Fig. 4 (right panel), the total polarizabilities in the main polarizations $\alpha_{e,c}^{\parallel}$ and $\alpha_{e,c}^{\perp}$ are given by

$$\alpha_{e,c}^{\parallel}(\omega) = \frac{\alpha_e^{cp}(\omega)}{1 - j\frac{k_0^2}{4}\alpha_e^{cp}(\omega)},$$

$$\alpha_e^{cp}(\omega) = \pi\varepsilon_0 a^2[\varepsilon_r(\omega) - 1] - \pi\varepsilon_0(b^2 - a^2)[\varepsilon_c(\omega) - 1] \quad (8)$$

and

$$\alpha_{e,c}^{\perp}(\omega) = \frac{\alpha_e^{co}(\omega)}{1 - j\frac{k_0^2}{4}\alpha_e^{co}(\omega)},$$

$$\alpha_e^{co}(\omega) = \pi b^2\varepsilon_0 \frac{b^2(\varepsilon_c - 1)(\varepsilon_c + \varepsilon_r) - a^2(\varepsilon_r + 1)(\varepsilon_c - \varepsilon_r)}{b^2(\varepsilon_c + 1)(\varepsilon_c + \varepsilon_r) - a^2(\varepsilon_c - 1)(\varepsilon_c - \varepsilon_r)}. \quad (9)$$

In Fig. 5, we show the complex polarizability of a coated nanorod consisting of a dielectric inner core with $\varepsilon_r = 2$ and a lossy plasmonic external shell whose permittivity is modeled through a Drude-type dispersion model, as a function of the normalized frequency ω/ω_p and $\beta = a/b$, assuming $\nu_c = 10^{-2}\omega_p$.

According to Eq. (9), at a given pair (ω_0, β) it may result $\alpha_{e,c}^{\perp}(\omega_0) \approx 0$, thus implying that no force is exerted on the object for that polarization. As in the spherical case, moderately high losses still allows to obtain the cancellation effect, as reported in Fig. 6.

For more complex geometries, as recently shown in Ref. 8, it is still possible to use conformal plasmonic coatings to effectively cloak the object, even in the case the scatterer shows a strong anisotropic electromagnetic response. Moreover, for moderately low dielectric contrasts, Rayleigh particles should exhibit polarizability values very close to the ones of an isovolumetric sphere, thus implying that similar considerations may be straightforwardly applied even for noncanonical shapes.

III. OPTICAL TORQUE FOR CLOAKED RAYLEIGH PARTICLES

Starting from Eq. (1), we can derive the intrinsic optical torque $\mathbf{\Gamma}$ acting on a small particle under Rayleigh approximation. For a dielectric object illuminated by an external field $\mathbf{E}_0$, the induced dipole $\mathbf{p}$ leads to an electric torque which, adding the radiation-reaction term to the static polarizability α_e^0, is given by[22,28]

$$\mathbf{\Gamma} = \frac{1}{2}\mathfrak{Re}\left[\mathbf{p} \times \left(\frac{\mathbf{p}}{\alpha_e^0}\right)^*\right]. \quad (10)$$

Consequently, the torque-vector magnitude is directly proportional to α_e^0 and all the considerations done in the previous sections still apply. Interestingly, since in the more gen-

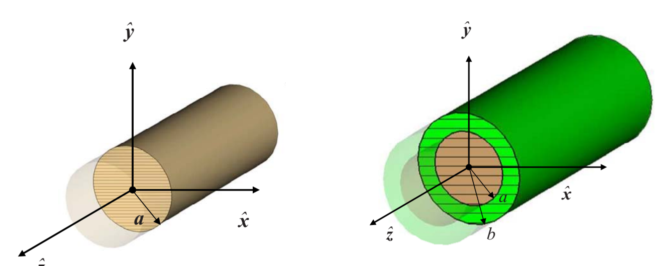

FIG. 4. (Color online) Cylindrical nanorod of radius a infinitely extended along its symmetry axis (left). Cylindrical nanorod of radius a infinitely extended along its symmetry axis and covered with a plasmonic shell of radius b.

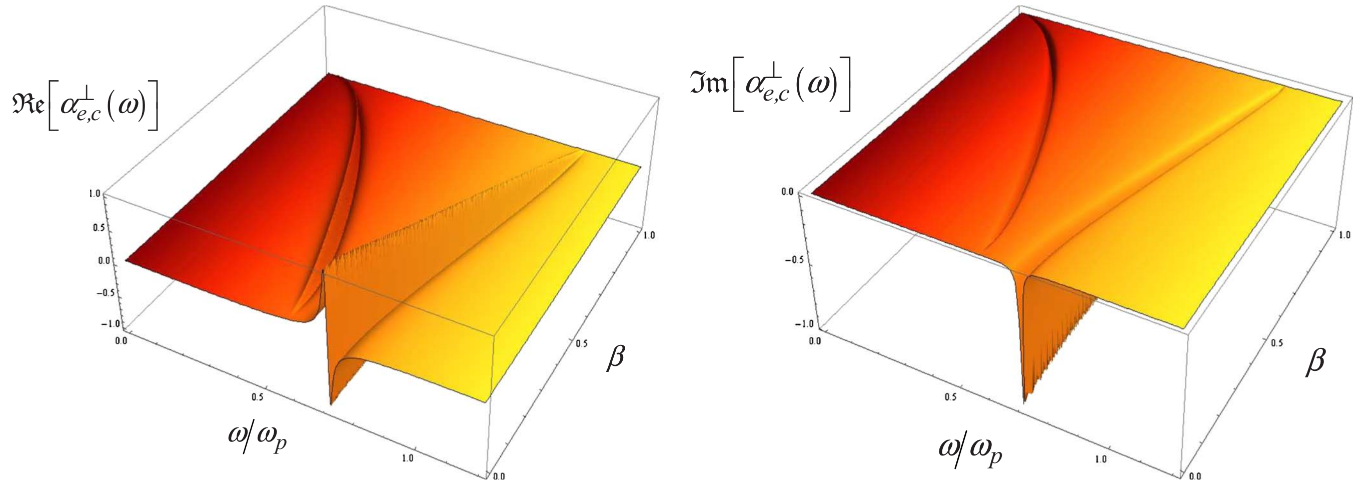

FIG. 5. (Color online) Normalized complex polarizability for a dielectric nanorod covered with a plasmonic coating, whose permittivity follows Drude dispersion (electric field orthogonal to the symmetry axis).

eral case the polarizability is a tensor and $\mathbf{p}=\underline{\boldsymbol{\alpha}}_e(\omega)\cdot\mathbf{E}_0$, the torque magnitude depends not only on the orientation of the impinging field but also on the elements of $\underline{\boldsymbol{\alpha}}_e(\omega)$. This implies that the stability of the configuration may change when coating an object with a plasmonic shell. In particular, it is possible to determine new equilibrium positions or to make stable an equilibrium position which results unstable for the bare particle.

Let us consider, for instance, the case of a dielectric cylinder of radius a and length L with the major axis aligned along the $\hat{z}$ direction. We may assume that the polarizability of such a particle is uniaxial, as

$$\underline{\boldsymbol{\alpha}}_e = \alpha_e^{zz}\hat{z}\hat{z} + \alpha_e^{t}\underline{\mathbf{I}}_t, \tag{11}$$

where the components of $\underline{\boldsymbol{\alpha}}_e$ may be derived either by interpolating functions[27] or by remembering that, for a finite length electrically thin cylinder, the SCS (and, consequently, the polarizability) is proportional to the one of the infinite case, through the effective length of the object.[8,21,29] Interestingly, expression (11) holds, in general, for any given elongated symmetrical particle. For instance, a prolate spheroid may be described by a closed-form uniaxial polarizability tensor,[27,29] for which it is well known that, when the impinging electric field is parallel to one of the three main axes, no torque is exerted. If the spheroid is made of a regular dielectric with $\varepsilon_r > 1$, the direction of the torque components is independent from the magnitude of ε_r, and, since $\alpha_e^{zz} > \alpha_e^{t}$, the equilibrium positions along the minor axes are unstable, so that the particle tends to align with the major axis along the impinging electric field direction.[29]

Since the relative amplitudes of the tensor elements determine the nature (stable or unstable) of the equilibrium positions by using the polarizabilities derived for the infinite length coated cylinder (see previous section), we may now find some regions in the dielectric plane described by $(\varepsilon_c, \varepsilon_r)$ such that unstable equilibrium positions for the bare particle may be turned to stable ones for the coated particle, and vice versa.

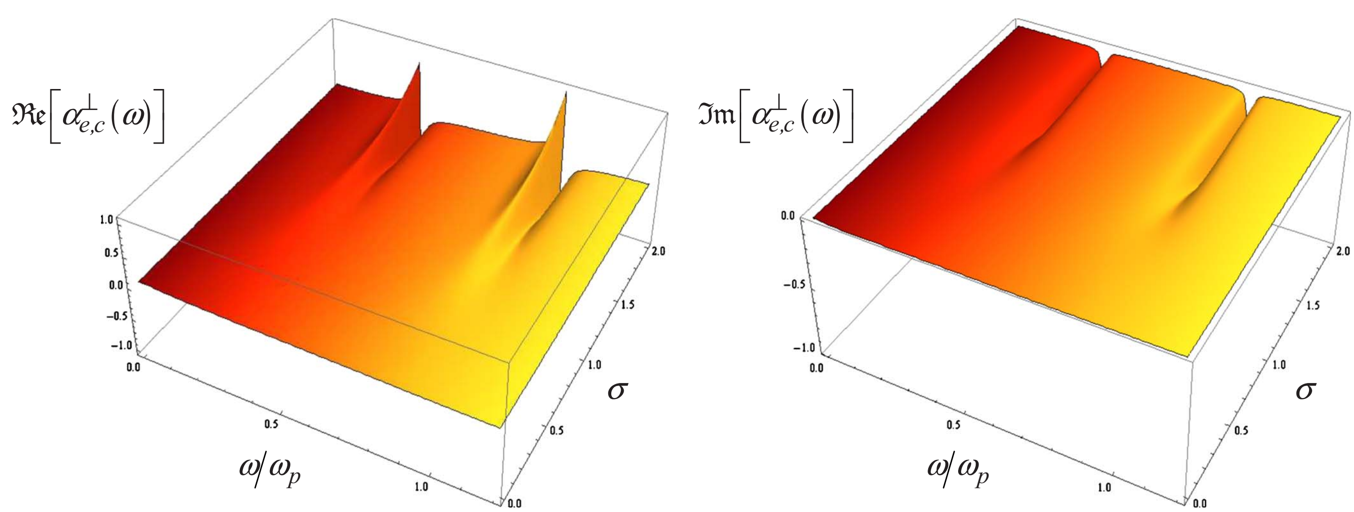

FIG. 6. (Color online) Normalized complex polarizability for a dielectric nanorod (electric field orthogonal to the symmetry axis) covered with a plasmonic coating whose permittivity follows Drude dispersion for $\beta \approx 0.67$ and assuming losses to vary according to $\nu_c = 10^{-\sigma}\omega_p$.

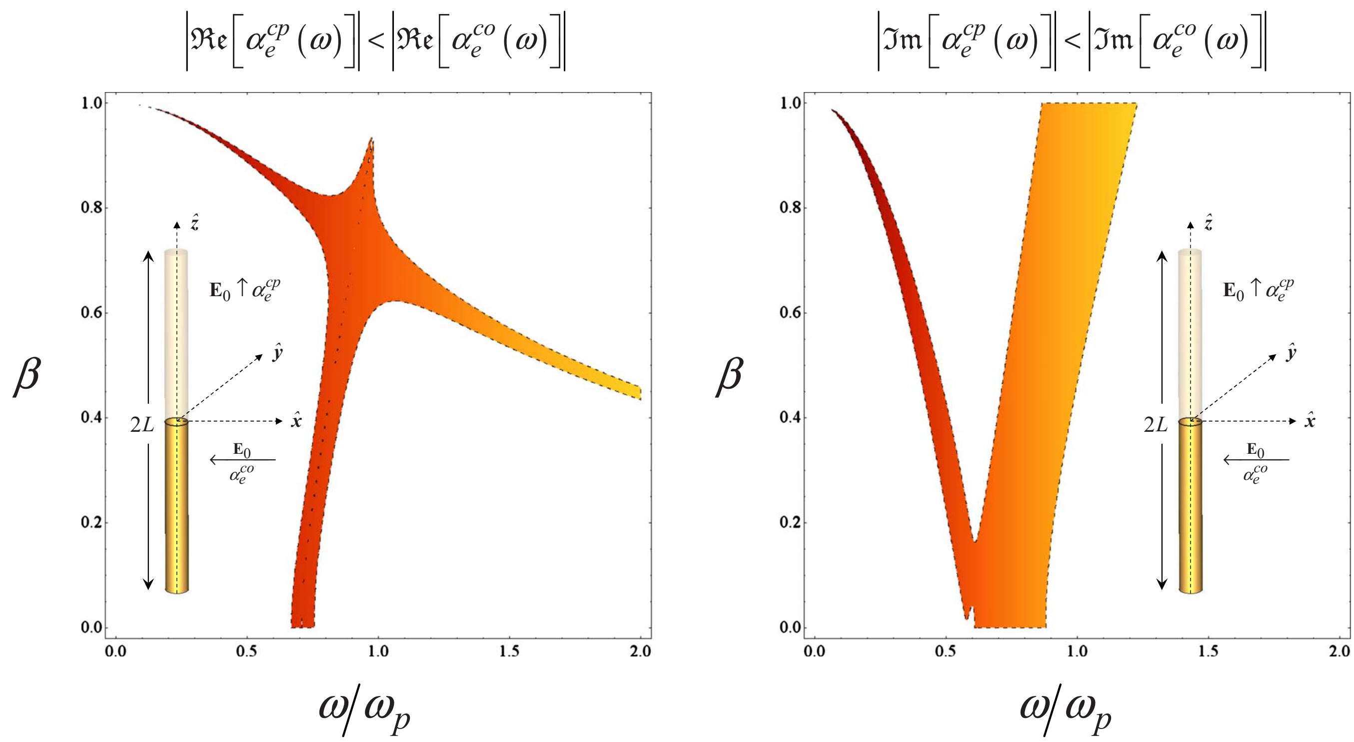

FIG. 7. (Color online) Region plots referred to the magnitude of the complex polarizability tensor elements of a dielectric nanorod with $\varepsilon_r=2$, covered by the same plasmonic shell used in Fig. 5. White areas represent the points at which the relations shown in the insets are not satisfied.

In Fig. 7, region plots, referred to the magnitude of the complex polarizability tensor elements of a dielectric cylindrical nanorod with $\varepsilon_r=2$ covered by the same plasmonic shell as in the previous example, are reported. The colored area in the two plots corresponds to the points for which $|\mathfrak{Re}[\alpha_e^{cp}(\omega)]| < |\mathfrak{Re}[\alpha_e^{co}(\omega)]|$ and $|\mathfrak{Im}[\alpha_e^{cp}(\omega)]| < |\mathfrak{Im}[\alpha_e^{co}(\omega)]|$, respectively, that is when both real and imaginary parts of the orthogonal polarizability are greater than the longitudinal ones.

From Fig. 7, it is possible to determine a proper parameter range in which the acting torque may be effectively controlled by the plasmonic behavior of the cover. In particular, when real and imaginary parts of the orthogonal polarizability are both greater than the ones of the longitudinal polarizability, the illuminated nanorod may experience an anomalous torque, leading to the alignment of the particle along the direction of the minor axis. This position, which for uncovered particles is usually an unstable one, turns out to be a stable equilibrium position in the case of the covered particle. This is only one of the possibilities offered by the new degrees of freedom introduced by plasmonic cloaking for this particular configuration. Further possibilities result also from the exploitation of the cover material dispersion, which may be used to switch equilibrium positions from stable to unstable ones, according to the plots in Fig. 7.

IV. OPTICAL FORCES ON A CLOAKED RAYLEIGH PARTICLE NEAR A DIELECTRIC HALF SPACE

So far we have considered only particles in free space. In order to show how the new possibilities offered by plasmonic cloaking can be applied also to more complex scenarios, we consider now the case of a Rayleigh particle placed near the interface of a dielectric half space and illuminated by an external field $\mathbf{E}_0$, as shown in Fig. 8.

In the quasistatic limit, the total electric field at the sphere position may be written as[26]

$$\mathbf{E} = \left[\underline{\mathbf{I}} - \frac{\alpha_e}{\varepsilon_0}(\omega)\underline{\mathbf{S}}\right]^{-1} \cdot \mathbf{E}_0, \tag{12}$$

where the tensor $\underline{\mathbf{S}}$ is the linear response of an electric dipole in presence of the half space (polarizability is assumed to be a constant but also anisotropic media may be considered simply by changing α_e when varying the polarization of the applied field). Assuming that the distance between the object and the surface is electrically small, in the long-wavelength limit $\underline{\mathbf{S}}$ is uniaxial, and its elements are related only to the

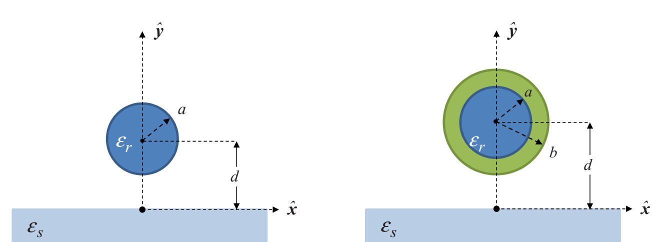

FIG. 8. (Color online) A Rayleigh spherical particle placed above a dielectric flat surface (left). Rayleigh spherical particle covered with a plasmonic shell placed above a dielectric flat surface (right).

relative distance from the particle and to the electric properties of the half space, that is to its relative permittivity ε_s, as[26]

$$\underline{\underline{S}} = \frac{1}{8d^3}\frac{\varepsilon_s - 1}{\varepsilon_s + 1}\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix} = \frac{1}{8d^3}\gamma\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{13}$$

being γ the Fresnel reflection coefficient, and d the distance of the surface from the center of the particle. As we can see from expression (13), when $d \to \infty$ we obtain the response of a particle in free space while for any other value of d, provided that it is smaller than the wavelength, we may evaluate the dipolar coupling. If the propagation vector lies in the xy plane, no force is exerted along $\hat{z}$, due to the symmetry of the problem. Consequently, if we consider a plane-wave impinging on the sphere with the electric field directed along $\hat{z}$, the force components are given by[26]

$$F_x = \frac{|E_{0z}|^2\varepsilon_0}{2}\mathfrak{Re}\left[\frac{8d^3\alpha_e(jk_{0x})^*}{8\varepsilon_0 d^3 + \gamma\alpha_e}\right],$$

$$F_y = \frac{\varepsilon_0|E_{0z}|^2}{2}\mathfrak{Re}\left[\frac{8d^3\alpha_e(jk_{0y})^*}{8\varepsilon_0 d^3 + \gamma\alpha_e} + \frac{12d^2|\alpha_e|^2\gamma}{|8\varepsilon_0 d^3 + \gamma\alpha_e|^2}\right] \tag{14}$$

while in the orthogonal polarization it is found

$$F_x = \frac{\varepsilon_0}{2}\mathfrak{Re}\left[4d^3\alpha_e(jk_{0x})^*\left(\frac{2\varepsilon_0|E_{0x}|^2}{8\varepsilon_0 d^3 + \gamma\alpha_e} + \frac{\varepsilon_0|E_{0x}|^2}{4\varepsilon_0 d^3 + \gamma\alpha_e}\right)\right],$$

$$F_y = \frac{|E_{0x}|^2\varepsilon_0}{2}\mathfrak{Re}\left[\frac{8d^3\alpha_e(jk_{0y})^*}{8\varepsilon_0 d^3 + \gamma\alpha_e} + \frac{12d^2|\alpha_e|^2\gamma}{|8\varepsilon_0 d^3 + \gamma\alpha_e|^2}\right] + \frac{|E_{0y}|^2\varepsilon_0}{2}\mathfrak{Re}\left[\frac{4d^3\alpha_e(jk_{0y})^*}{4\varepsilon_0 d^3 + \gamma\alpha_e} + \frac{6d^2|\alpha_e|^2\gamma}{|4\varepsilon_0 d^3 + \gamma\alpha_e|^2}\right]. \tag{15}$$

In the case of a covered sphere, we are able to reduce the acting forces (mainly related to the radiation pressure), in the same way as for the configurations considered in the previous sections. In fact, by inserting $\alpha_e^{cs}(\omega)$ from Eq. (5) in Eqs. (14) and (15), we easily obtain by inspection that all the force components vanish if both the real and the imaginary parts of the polarizability of the coated spherical particle $\alpha_e^{cs}(\omega)$ vanish.

Interestingly, a similar approach may be used also for cylindrical particles, simply by substituting the proper polarizability $\alpha_e^{\parallel}(\omega)$ or $\alpha_e^{\perp}(\omega)$, according to the polarization of the external field, and by using for $\underline{\underline{S}}$ the form

$$\underline{\underline{S}} = \frac{1}{2d^2}\frac{\varepsilon_s - 1}{\varepsilon_s + 1}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{16}$$

It is well known that, when a dielectric particle is close to the interface, it may experience an attractive force directed in the opposite direction of propagation of an normally incident illuminating field, due to the interaction of the dipole with its own reflected evanescent field.[26] When coating the particle, the force does not depend only on the distance d anymore but it can be changed also through the electric permittivity of the cover $\varepsilon_c(\omega)$.

Let us consider, for instance, the case of a plane wave with the electric field directed along $\hat{z}$ and $k_0 = k_y$, normally impinging on the planar surface separating the vacuum and the material. In the case of a spherical particle placed in the vacuum half space at a distance d from the surface, Eq. (14), after some manipulation, simplifies as

$$F_y = \frac{\varepsilon_0|E_0|^2 64d^3}{2|8\varepsilon_0 d^3 + \gamma\alpha_e^{cs}|^2}\left\{\frac{k_0}{\varepsilon_0}\mathfrak{Im}[\alpha_e^{cs}] + \frac{3\gamma|\alpha_e^{cs}|^2}{16d^4}\right\} = \frac{\varepsilon_0|E_0|^2 64d^3}{2|8\varepsilon_0 d^3 + \gamma\alpha_e^{cs}|^2}\left(\sigma_t + \frac{3\gamma|\alpha_e^{cs}|^2}{16d^4}\right). \tag{17}$$

In this expression, we straightforwardly recognize the contribution given by the total SCS σ_t of the particle while the last term is due to the interaction of the particle with the surface. The zeroes of F_y occur then at

$$\sigma_t = -\frac{3\gamma|\alpha_e^{cs}|^2}{16d^4} \tag{18}$$

while the direction of the force is given by $\sin[\sigma_t + 3\gamma|\alpha_e^{cs}|^2/16d^4]$. From expression (18) we may conclude that, for a regular dielectric lossless substrate ($\gamma > 0$), the zeroes of F_y are determined by negative values of σ_t. Such a condition may be achieved if also the imaginary part of the polarizability is negative, and this may happen through a proper choice of the plasmonic properties of the shell surrounding the particle, as described, for instance, in the example reported in Figs. 2 and 3. Similar conclusions may also apply for the cylindrical case.

From the previous analysis, it is clear that the light force upon a particle on a dielectric plane can be controlled through the distance d and the dispersive behavior of the material composing the shell. A suitable design may, thus, allow to engineer the small scatterer response with interesting applications in particle trapping and manipulation.

V. CONCLUSIONS

In this paper, we have proposed a theoretical analysis combining optical force calculation to the electromagnetic cloaking formulation, in order to gain further physical insights and explore new possibilities for the manipulation of nanoparticles. First, we have shown that the same conditions used to make spherical and cylindrical nanoparticles invisible to the electromagnetic field by using the scattering cancellation approach, can be straightforwardly employed also to minimize gradient and scattering optical forces exerted by the illuminating field on the same covered nanoparticles. These results have been verified through proper full-wave simulations considering properly both dispersion and losses of the plasmonic materials used to design the cloaks. We have also extended our speculation to the case of optical torques exerted on spheroidal and cylindrical Rayleigh particles, deriving the conditions to obtain stable equilibrium positions. This investigation led to the interesting result that, anomalously, the usual unstable equilibrium positions of uncovered particles may turn out to be stable ones when properly covering the particles. The additional degrees of freedom introduced by the plasmonic cloaking may be of interest in many nanotechnology applications. Finally, in order to apply the proposed theoretical speculations to more complex cases, we have derived the conditions for minimizing optical forces exerted on a cloaked Rayleigh particle placed above a dielectric half space. Such results may be straightforwardly extended to typical configurations used in several application fields, to obtain, for instance, improved biological sensing, optical trapping, and probing.

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